EFFECTS OF PARASITIC CONVECTION ON THE OPTIMAL DIFFUSION COLUMN WORKING IN THE SAMPLING MODE

A. V. Suvorov and G. D. Rabinovich

UDC 621.039.341.6

Optimization theory is discussed for a thermal-diffusion column working in sampling mode with parasitic convection.

It has been shown [1] that any temperature asymmetry around the perimeter of a thermal-diffusion column results in parasitic convection; a correction has been applied for this adverse factor [2] for the conditions of zero tapoff.

The physical and mathematical models of [2] are used here to extend the theory of parasitic convection to the sampling mode; the equations describing the steady state then contain not only a term representing the parasitic effect σ_p, but also a term σ corresponding to product tapoff:

$$\frac{1}{2}\left(Hc'(1-c')-K\frac{dc'}{dz}\right)+\left(\frac{\sigma}{2}+\sigma_p\right)c'=\left(\frac{\sigma}{2}+\sigma_p\right)c'_e,$$
$$\frac{1}{2}\left(Hc''(1-c'')-K\frac{dc''}{dz}\right)+\left(\frac{\sigma}{2}-\sigma_p\right)c''=\frac{\sigma}{2}c''_e-\sigma_p c'_e. \tag{1}$$

These equations are written for two regions in the thermal-diffusion column that differ in mixture density on account of the temperature asymmetry around the perimeter, where the parasitic circulation coincides in sense with the useful tapoff and is there given a positive value, whereas in the other region it is given a negative value.

We introduce the dimensionless variables

$$y=Hz/K;\quad \varkappa=\sigma/H;\quad \varkappa_p=2\sigma_p/H \tag{2}$$

to convert (1) to the form

$$\frac{dc'}{dy}-(1+\varkappa+\varkappa_p)c'+c'^2=-(\varkappa+\varkappa_p)c'_e,$$
$$\frac{dc''}{dy}-(1+\varkappa-\varkappa_p)c''+c''^2=\varkappa_p c'_e-\varkappa c''_e. \tag{3}$$

We solve (3) subject to the initial conditions

$$c'|_{y=0}=c^{*}|_{y=0}=c_0 \tag{4}$$

which for $y = y_e$ take the form

$$\operatorname{th} b'\frac{y_e}{2}=\frac{b'(c'_e-c_0)}{c'_e+c_0-2c'_e c_0-\varkappa(1+u)(c'_e-c_0)}, \tag{5}$$

$$\operatorname{th} b''\frac{y_e}{2}=\frac{b''(c''_e-c_0)}{c''_e+c_0-2c''_e c_0-\varkappa[c''_e-c_0-u(2c'_e-c''_e-c_0)]}, \tag{6}$$

where

$$u=\frac{\varkappa_p}{\varkappa};\quad b'=\sqrt{[1+\varkappa(1+u)]^2-4\varkappa(1-u)c'_e}; \tag{7}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 3, pp. 471-476, September, 1978. Original article submitted July 25, 1977.

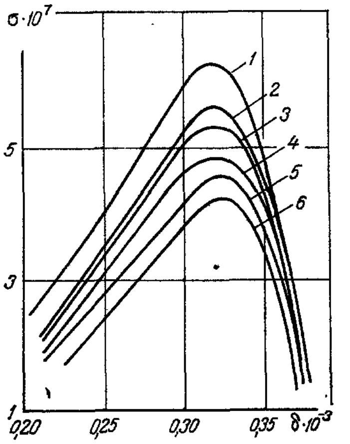

Fig. 1

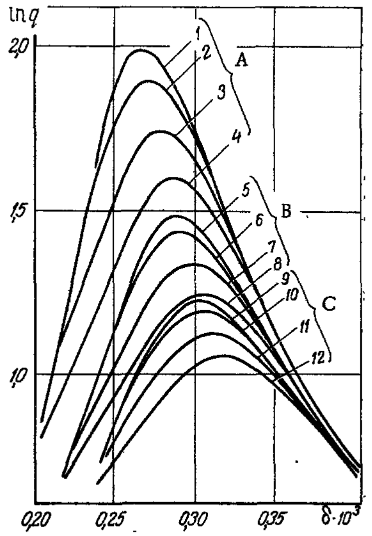

Fig. 2

Fig. 1. Effects of δ (m) on σ (kg/sec) for $H^* = 2 \cdot 10^4$ kg/sec·m^3, $y_e^* = 2 \cdot 10^{10}$ m^4: 1) $c_0 = 0.7$, $u = 0$; 2) 0.7 and 0.6; 3) 0.5 and 0; 4) 0.5 and 0.6; 5) 0.5 and 0.9; 6) 0.2 and 0.6.

Fig. 2. Dependence of $\ln q$ on δ (m) for $c_0 = 0.5$, $H^* = 2 \cdot 10^{-4}$ kg/sec·m^3, $y_e^* = 2 \cdot 10^{-10}$ m^4; A) $\sigma = 2 \cdot 10^{-7}$ kg/sec; B) $\sigma = 3 \cdot 10^{-7}$ kg/sec; C) $\sigma = 4 \cdot 10^{-7}$ kg/sec; 1, 5, 9) $u = 0$, 2, and 6; 10) $u = 0.3$, 3, and 7; 11) $u = 0.6$, 4, and 8; 12) $u = 0.9$.

$$b'' = \sqrt{[1 + \varkappa(1 - u)]^2 - 4\varkappa(c_e'' - uc_e')} \,. \tag{7}$$

From (5) and (6) we can derive c_e' and c_e'', whose values can be used in determining the concentration at the positive end of the column:

$$c_e = \frac{1}{2}(c_e' + c_e''). \tag{8}$$

The quantities y_e and $\varkappa$ appearing in (5) and (6) are functions of the gap δ, which follows from the definition of (2) if we substitute their values in place of H and K_c (see notation):

$$\begin{aligned} y_e &= \frac{HL}{K} = 504\,\frac{\alpha\eta DL}{\rho g\beta\delta^4} = \frac{y_e^*}{\delta^4}, \\ \varkappa &= \frac{6!\,\eta\bar{T}\sigma}{\alpha\rho^2 g\beta\delta^3(\Delta T)^2 B} = \frac{\sigma}{H^*\delta^3}. \end{aligned} \tag{9}$$

Here the quantities with asterisks are independent of the gap. Therefore, c_e in (8) should be dependent on six parameters: c_0, u, σ, H, y_e^*, and δ. If we know the physical characteristics of the mixture, the working parameters of the column, and the dimensions such as the length and perimeter, then the parameters that otherwise determine the working conditions are u, σ, and δ. By analogy with [3], we proceed as follows: We have to determine values of δ such that σ is maximized for a given degree of separation q in the presence of variable parasitic convection.

The calculations were performed on a Minsk-22 computer; Fig. 1 shows results for the case $\ln q = 1$.

This indicates that the optimum gap (that which provides the best output) is virtually independent of the initial concentration and of the parasitic effect represented by u in (7) for a given degree of separation, while the output from the column in the range $\delta > \delta_{opt}$ is more sensitive to any change in the gap than it is in the range $\delta < \delta_{opt}$; therefore, the latter is to be considered as the region of more stable operation. If, on the other hand, the output is to be constant, then the optimum gap, which corresponds in that case to maximum separation, shifts to smaller δ_{opt} as the parasitic-convection parameter decreases and as the tapoff is reduced, as Fig. 2 shows. This results is in agreement with previous ones [3] for the limiting cases $c \ll 1$ and

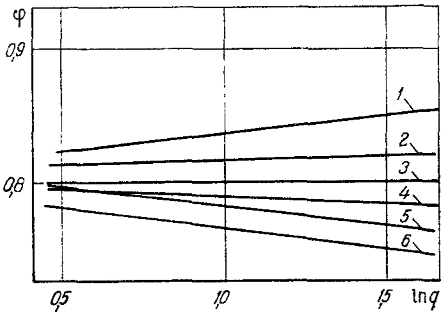

Fig. 3. Graphs for φ as a function of ln q: 1) $c_0 = 0.7$, u = 0; 2) 0.5 and 0; 3) 0.7 and 0.3; 4) 0.5 and 0.3; 5) 0.2 and 0; 6) 0.2 and 0.3.

$1-c \ll 1$. The effects of the tapoff are more important than those of parasitic convection. For example, increase in the tapoff by a factor 2.5 implies that the gap must be increased by 20% for the parameters H^*, y_e^*, and c_0, whereas increase in u from 0.3 to 0.9, i.e., by a factor 3, alters the optimum gap by only 4-8%.

Further analysis of the results from (5) and (6) reveals the effects of the parasitic convection in tapoff mode, particularly as regards the energy-utilization factor φ, which is defined as the ratio of the heat consumption per unit length for a column with the optimum gap to the same for an ideal stage with the same gap in the zero section, i.e., in accordance with [3] by

$$\varphi = \frac{Q_{\text{ep}}^{\text{id}}}{Q_{\text{ep}}} = \frac{40}{7}\,\frac{\varkappa_{\text{opt}}}{y_{\text{e,opt}}}\,\frac{H_{\text{opt}}^2\delta_{\text{op.}}}{K_{\text{opt}}B}\left(\frac{\bar{T}}{\alpha\Delta T}\right)^2\frac{V(c_e,\ c_0)}{\rho D}\,. \tag{10}$$

We substitute for H_{opt} and K_{opt} and use the following value functions [4]:

$$V(c_0,\ c_e) = \frac{(c_e - c_0)(1-2c_0)}{c_0(1-c_0)} - (1-2c_e)\ln q,$$

to obtain in place of (9) that

$$\varphi = 4\,\frac{\varkappa_{\text{opt}}}{y_{\text{e, opt}}}\,V(c_e,\ c_0). \tag{11}$$

Figure 3 shows results for the values of H^* and y_e^* used in the previous two cases; the effects of the parasitic convection on the energy performance factor increase with the degree of separation and with the initial concentration of the target component. However, the performance of a column of constant cross section is always higher for a given u and increases with c_0 [3], which is important in the use of thermal diffusion to purify substances from trace components. This shows that parasitic convection causes the separation to deviate appreciably from the theoretical prediction for an ideal model, which involves the assumption of perfectly isothermal working surfaces.

It is therefore of interest to examine the effects arising from the useful tapoff on the deviation of the separation factor from the theoretical value; a measure of the deviation is $\ln q/\ln q^*$, where q and q^* are the degrees of separation in a column with and without parasitic convection, respectively.

Figure 4 shows that the performance approaches the theoretical value for an ideal column as the tapoff increases for a given value for the parasitic convection, and the actual value may approach 95% of the theoretical one. This explains some results [5] on the separation of heptane–benzene mixtures, which indicate that the agreement with theory is good only in tapoff mode, whereas the deviation from theory was very considerable when there was no tapoff. In fact, Fig. 4 shows that $\ln q/\ln q^* = 0.39$ without tapoff if $\varkappa_p = 0.5$, whreas $\ln q/\ln q^* = 0.96$ in the tapoff mode with $\varkappa = 1$. Threfore, the difference is very substantial. If the conditions are such that $\ln q/\ln q^* \to 1$, a thermal-diffusion column can be used to advantage to determine the Soret coefficient, as has been shown elsewhere [6]. This method has advantages over the nonstationary method [7] because it does not require very precise measurement of small concentration shifts.

To gain a clearer idea of the design specifications for a column for this purpose we consider the case corresponding to curve 3 of Fig. 4; the parasitic tapoff factor is [2]

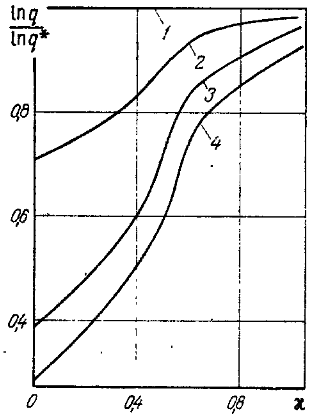

Fig. 4. The ϰ dependence of $\ln q/\ln q^*$ for $c_0 = 0.5$, $H^* = 2 \cdot 10^4$ kg/sec·m^3, $y_e^* = 2 \cdot 10^{10}$ m^4: 1) $\varkappa_p = 0$; 2) 0.25; 3) 0.5; 4) 0.7.

$$\varkappa_p = 15\bar{T}(\delta T)/\alpha(\Delta T)^2, \tag{12}$$

and in this case $\varkappa_p = 0.5$, while $\alpha \approx 1$ for many mixtures, so we put $\Delta T = 100°$, $\bar{T} = 330°$K, to obtain that $\delta T = 1°$; on the other hand, it has been shown [8] that

$$\delta T = \frac{\varepsilon \Delta T}{\delta}\psi, \tag{13}$$

where the quantity $\psi \leq 1$ is dependent on the heat-transfer conditions at the surfaces, and the value for this is taken as 0.2 for the purposes of estimates. Then $\varepsilon = 0.015$ mm, i.e., the precision in manufacturing the column must be rather high for these conditions.

Also, we have to determine the useful tapoff, which corresponds to the right-hand part of the curves in Fig. 2, and the value must be such that $\ln q/\ln q^*$ differs little from 1.

Therefore, Figs. 1-4 can be used in designing a column for analytical or technological purposes, although it must be borne in mind that changes in c_0, H^*, and y_e^* can affect the results to a certain extent.

NOTATION

$H = \alpha g \rho^2 \beta \delta^3 (\Delta T)^2 B / 6! \eta \bar{T}$;
$K_c = g^2 \rho^3 \beta^2 \delta^7 (\Delta T)^2 B / 9! \eta^2 D$;

c	is the mass concentration;
σ	is the useful tapoff ratio;
σ_p	is the parasitic tapoff ratio;
z	is the vertical coordinate;
y	is the dimensionless coordinate of (2);
$\varkappa$, $\varkappa_p$	are the dimensionless useful and parasitic ratios in (2);
u	is the quantity in (7);
y_e^*, H^*	are the quantities defined in (9),
δT	is the temperature asymmetry (temperature difference between hot and cold sections);
ε	is the eccentricity in (13);
q, q^*	are the separation factors in the presence and absence of parasitic convection, respectively;
α	is the thermal-diffusion constant;
B, δ	are the perimeter and width of gap, respectively;
η, D, β	are the dynamic viscosity, diffusion coefficient, and thermal-expansion coefficient, respectively;

ΔT is the temperature difference between working surfaces.

Indices

e is the positive end;
p is the parasitic end;
0 is the initial value;
' is the concentration referred to region 1;
'' is the concentration referred to region 2.

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DIFFERENTIAL TRANSFER EQUATIONS FOR MULTIPHASE, MULTICOMPONENT MEDIA

V. I. Pavlov

UDC 532.529:541.124

Transfer equations of mass and momentum are obtained for single-phase, single-component and for multiphase, multicomponent media with account taken of substance change. Similarity criteria for these media are analyzed.

Investigations of transfer processes in multiphase, multicomponent media are topical problems in view of their wide application.

A considerable part of the investigations was extended in [1]. Further development was carried out in [2, 3]. In [2], transfer processes in a two-phase multicomponent medium are described and a thermodynamic analysis is carried out. The adopted assumptions, however, limit the range of applications for the obtained equations.

In the present article the transfer of mass and momentum in a multiphase, multicomponent (n, m) medium is described in accordance with the concepts of Sedov [4, 5], fruitfully applied by him to develop the mechanics of the multiphase media [1].

Let us consider a volume element of the medium with considerably smaller dimensions than those of the phase elements.

It is assumed that the transfer of a substance (mass, momentum) within a separate component, phase, or mixture can be described similarly as for a solid medium, but now the substance transport between the phases or components in this continuum is also taken into account.

In contrast to other investigations [2, 3], no restrictions are imposed as regards the effect of the shape, the number of phases (the number of phases $n \geq 1$), or the number of components (the number of components $m \geq 1$). The phases may be continuous or discrete. The elements of any phase may interact either with the

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Sofia, Bulgaria. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 3, pp. 477-487, September, 1978. Original article submitted February 2, 1977.